Period 3, Sept 30, 2024

3. $h(t) = \sqrt[3]{2t+7} + e^{t^2}$

$y = \sqrt[3]{2T+7} \Rightarrow y = \sqrt[3]{u} = u^{\frac{1}{3}}$

$u = 2T+7$ $\quad \frac{dy}{du} = \frac{1}{3\sqrt[3]{(u^2)}}$

$\frac{du}{dT} = 2$

$\frac{\cancel{du}}{dT} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dT}$

$2 \cdot \frac{1}{3\sqrt[3]{u^2}} = \frac{2}{3\sqrt[3]{(2T+7)^2}}$

$y = e^{T^2} \Rightarrow y = e^u$

$u = T^2$ $\quad \frac{dy}{du} = e^u \cdot u' = e^u \cdot 1$

$\frac{du}{dT} = 2T$

$\frac{\cancel{du}}{dT} \cdot \frac{dy}{\cancel{du}} = \frac{dy}{dT}$

$2T \cdot e^u = 2Te^{T^2} = \frac{dy}{dT}$

$h(T) = \sqrt[3]{2T+7} + e^{T^2}$

$h'(T) = \frac{2}{3\sqrt[3]{(2T+7)^2}} + 2Te^{T^2}$

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5. $y = \sin(\cos x)$ at $x = \frac{\pi}{6}$.

$y = \sin(\cos x)$

$y = \sin u$

$u = \cos x$

$\frac{du}{dx} = -\sin x$ $\quad \frac{dy}{du} = \cos u$

$\frac{\cancel{du}}{dx} \cdot \frac{dy}{\cancel{du}} = -\sin x \cdot \cos u = -\sin x \cos(\cos x) = \frac{dy}{dx}$

$= -\sin\frac{\pi}{6} \cos(\cos\frac{\pi}{6}) = -\frac{1}{2} \cdot \cos\left(\frac{\sqrt{3}}{2}\right) = -\frac{1}{2} \cdot (.6478)$

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6. Let $r(x) = f(h(x))$, where $h(2) = 3$, $h'(2) = 4$, $f'(2) = 2$, $f(2) = 1$, $f'(3) = 6$, find $r'(2)$.

$r(x) = F(h(x))$

$r'(x) = F'(h(x)) \cdot h'(x)$

$r'(2) = F'(h(2)) \cdot h'(2) \Rightarrow F'(3) \cdot 4 = 6 \cdot 4$

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10. (Challenge/Optional) Find the derivative of $y = cos^2\sqrt{\sin(\tan(\pi x))}$

$u = \pi x$ $\quad y = \cos^2\sqrt{\sin(\tan u)} \Rightarrow y = \cos^2\sqrt{\sin L} \Rightarrow y = \cos^2\sqrt{K} \Rightarrow y = \cos^2 r \Rightarrow y = P^2$

$\frac{du}{dx} = \pi$ $\quad L = \tan u$ $\quad \frac{dL}{du} = \sec^2 u$ $\quad K = \sin L$ $\quad \frac{dK}{dL} = \cos L$ $\quad r = \sqrt{K} = K^{\frac{1}{2}}$ $\quad \frac{dr}{dK} = \frac{1}{2\sqrt{K}}$ $\quad P = \cos r$ $\quad \frac{dP}{dr} = -\sin r$ $\quad \frac{dy}{dP} = 2P$

$$\frac{du}{dx} \cdot \frac{dL}{du} \cdot \frac{dK}{dL} \cdot \frac{dr}{dK} \cdot \frac{dP}{dr} \cdot \frac{dy}{dP} = \frac{dy}{dx}$$

$$\pi \cdot \sec^2 u \cdot \cos L \cdot \frac{1}{2\sqrt{K}} \cdot -\sin r \cdot 2P$$

$$\pi \cdot \sec^2 \pi x \cdot \cos(\tan \pi x) \frac{1}{2\sqrt{\sin(\tan \pi x)}} \cdot -\sin\sqrt{\sin(\tan \pi x)} \cdot 2\cos\left(\sqrt{\sin(\tan \pi x)}\right)$$

V_0 initial velocity, starting speed, thrown up/down

$$S(T) = \frac{1}{2}aT^2 + V_0 T + S_0 = \text{position}$$

a, V_0, S_0 are constants

acceleration

$9.8\ m/s^2 = 32\ FT/sec^2$ gravity

initial position where is particle at $T = 0$

$\frac{\Delta y}{\Delta x} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{dy}{dx}$

$y = \frac{1}{2}aT^2 + V_0 T + S_0$

$\frac{dy}{dT} = \frac{1}{\cancel{2}} a \cdot \cancel{2} T^{2-1} + V_0 \cdot 1 \cdot T^{1-1} + 0 = aT + V_0$

$S(T) = \text{Position}$

$S'(T) = V(T) = \text{Velocity}$

$S''(T) = V'(T) = \text{acceleration}$

$\frac{\text{Change in Position}}{\text{Change in Time}} = \text{Velocity}$

Speeding up: velocity and acc have the same (both positive or both negative)

$\frac{dy}{dT} = aT + V_0$

$\frac{d^2y}{d^2T} = a = \frac{\text{Change in Velocity}}{\text{Change in Time}} = -32\ FT/sec^2$

Slowing Down: velocity and acc have differant signs (one is positive the other is negative)

9. The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s is measured in meters.

a) Find the velocity of the particle at time t = 2. Indicate units of measure.

$$S'(T) = 3T^2 - 12T + 9 = 3[T^2 - 4T + 3] = 3(T-3)(T-1)$$
$$S'(2) = 3(2-3)(2-1) = 3 \cdot -1 \cdot 1 = -3 \text{ m/s}$$

b) When is the particle at rest? Justify your response.

$V(T) = 0$

The particle is at rest when velocity = 0

$S'(T) = V(T) = 3(T-3)(T-1)$

$V(T) = 0$ when $T = 3$ or 1 sec

$T = 3, 1$

$V(3) = 3(3-3)(3-1) = 3 \cdot 0 \cdot 2 = 0$ or $V(T) = 3(1-3)(1-1) = 3 \cdot -2 \cdot 0 = 0$

c) Determine whether the particle is speeding up or slowing down at time t = 4. Justify your response.

speeding up: $V(T)$ and $a(T)$ have same sign

slowing down: $V(T)$ and $a(T)$ have different signs

$V(T) = 3T^2 - 12T + 9 = 3(T-3)(T-1)$ $\quad V(4) = 3(4-3)(4-1) = 3 \cdot 1 \cdot 3 = +9$

$a(T) = V'(T) = 6T - 12 = 6(T-2)$ $\quad a(4) = 6(4-2) = 6 \cdot 2 = +12$

Particle is speeding up because $V(4)$ and $a(4)$ are both positive

1. A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec (about 86 km/h) reaches a height of $s(t) = 24t - 0.8t^2$ meters in t seconds.

a) Find the rock's velocity and acceleration as functions of time. (The acceleration in this case is the acceleration of gravity on the moon). $S'(T) = V(T)$ and $S''(T) = V'(T) = a(T)$

b) How long did it take the rock to reach its highest point? $V(T) = 0$ Find Time, Plug Time into Position Function

c) How high did the rock go?

d) When did the rock reach half its maximum height? Find half of Max height $\frac{1}{2}$ Max height $= S(T)$ Find T

e) How long was the rock aloft?

$S(T) = 0$ Takeoff

$S(T) = 0$ Landing